Optics Laboratory Work:
Fabry Perot Interferometer

ROOM MED 2 1117

Discovery Learning Lab

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1. Objectives

The objectives of this optics laboratory session are:

- Learn how to operate and align a Fabry-Perot interferometer under different conditions
- Learn how to measure the characteristics of such an interferometer
- Analyze the fringe pattern for the plan mirror configuration with a camera

It is essential that you prepare this lab session in advance by reading through the lab notes and selected reference documents. The tasks denoted with a symbol "☖" are short questions that require preparation before the lab (a handwritten draft is sufficient). In addition, there will be an oral examination during the first session about the methods described in these notes.

Make sure to take notes and save images during the lab session (in your Lab notebook), so that you can include this material in your final report.

If you need more information about certain topics of this lab session, feel free to consult the references given at the end of this document.

2. Safety

The HeNe laser used in this lab provides a cw (continuous wave) output of 2.5 mW. Please, use appropriate laser safety goggles in order to avoid damage to your eyes. Use the key switch at the laser power supply due to high voltage risks (20 kV).

You should observe the following rules:

- Never look into a direct or reflected laser beam.
- Do not wear watches, rings or any other object that can reflect the laser beam.
- Keep eyes level higher than laser level, i.e. avoid sitting on the chair while the laser is on.
- Laser safety goggles are available. Use them for safety.

The piezo-actuator is operated at 150 V, its power supply may cause an electric shock. Do not remove the associated BNC cable from the back of the control unit PTC 1000!

3. The Fabry-Pérot Interferometer

3.1. Introduction

The Fabry-Pérot interferometer (also called Fabry-Pérot cavities) was invented in 1897 by Charles Fabry and Alfred Pérot. In contrast to other, more conventional types like the Michelson or Mach-Zehnder interferometer, the Fabry-Pérot arrangement acts as an optical resonator which may result in an extremely high spectral resolving power $\lambda/\Delta\lambda$ up to $\sim 10^7$ for optical wavelengths λ. In this way, state-of-the-art Fabry-Pérot cavities may exceed the resolution of classical diffraction gratings by a factor of ~ 100 and provide an irreplaceable tool in particular for studies of the hyperfine structure in atomic spectra. They are even more important in laser physics for free space lasers.


Figure 1: Charles Fabry (1867-1945), left, and Alfred Perot (1863-1925), right, were the first French physicists to construct an optical cavity for interferometry. Figure is taken from Wikipedia.

3.2. The plane mirror resonator

We consider two plane mirrors with equal reflectivity $0 < r < 1$ in a parallel arrangement separated by a distance d from each other. A plane wave with an amplitude E_e falls onto the system with an angle of incidence α. At each mirror surface, the beam is partially reflected (amplitude coefficient $r < 1$) and transmitted. The scheme is sketched in Fig. 2.

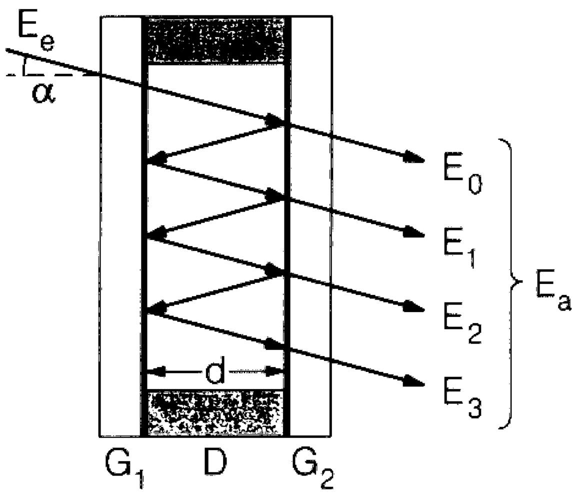

Figure 2: Multiple-beam interference at two parallel surfaces G1 and G2, separated by a distance d and filled with a dielectric medium D. Figure is taken from Web.

The phase difference $\Delta\varphi$ between two adjacent beams E_i and E_{i+1} depends on the mirror distance d, the refractive index n of the dielectric medium between the mirrors and the incident angle as:

$$\Delta\varphi = \frac{4\pi}{\lambda} n\, d \cos\alpha. \tag{1}$$

with λ the wavelength of light. We assume n = 1 from now on because we work with an air-filled resonator. The output electrical field E_a may be written as the coherent superposition of all amplitudes E_i with $0 \le i < \infty$, since an effectively infinite number of interfering waves is assumed for mirror with a reflectivity that approaches unity ($r \approx 1$). The total transmitted amplitude E_a becomes

$$E_a = \sum_{n=0}^{\infty} E_n = t^2 E_e \sum_{n=0}^{\infty} r^{2n} e^{-in\Delta\varphi} = \frac{t^2 E_e}{1 - r^2 e^{-i\Delta\varphi}}, \tag{2}$$

where the initial amplitude E_e is transmitted twice: $E_0 = t^2 E_e$. We use the relations $T = t^2$ and $R = r^2$ for the intensity transmission and reflection, respectively. If there is no absorption, we have $T + R = 1$ and the transmitted intensity is given as:

$$I_T = I_e \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\Delta\varphi/2)}, \tag{3}$$

for an incident intensity $I \propto |E|^2$. For an incidence angle that vanishes ($\alpha \to 0$) The phase difference (1) determines the transmitted wavelengths λ_m in the so called m^th^ order and one finds the relation

$$m\,\lambda_m = 2d \qquad \text{with } m \ge 1, \tag{4}$$

This condition defines an optical resonator, tuned by an adjustable mirror distance *d*. The integer number m represents the number of half wavelengths that can be found in the resonator as standing waves (compare Fig 5. for an illustration).

The transmitted intensity is very often expressed in a different way. We may write Eq. (3) in an alternative form and normalized to the incident intensity I_e.

$$T = \frac{1}{1 + (2/\pi)^2 \mathcal{F}_R^2 sin^2(\Delta\phi/2))} \qquad \text{with} \qquad \mathcal{F}_R = \pi\frac{\sqrt{R}}{1-R} \tag{5}$$

This transmission function is plotted in Fig. 3.

The *finesse* F_R quantifies the optical quality of the resonator made of perfectly polished and adjusted set of plane mirrors. However, even well-aligned resonators with plane mirrors are severely restricted to F<50 because of the residual surface roughness of even high-precision mirrors. The contribution of mirror irregularities which cause unwanted phase shifts at each reflection may be denoted by F_q. From an experimental point of view, the finesse describes the ratio between the free spectral range (FSR) δν (distance between two principle resonance) and the spectral resolution Δν of the instrument (specific to the measurement conditions) in the frequency domain:

$$F \equiv \delta\nu/\Delta\nu \qquad \text{with } \delta\nu \leq c/(2d), \tag{6}$$

where $c=c_0/n$ denotes the velocity of light in medium with refractive index n and c_0 the vacuum speed of light. For plane mirrors (air n=1) the free spectral range (FSR) $\delta\nu$ is given by $\delta\nu = c/(2d)$. $\Delta\nu$ is usually defined as the full width at half maximum (FWHM) of the instrument function or resonance. The finesse F also indicates the effective number of interfering beams within the cavity. If the number of interfering beams is increased the finesse increases and the resolution gets better. This is in analogy to grating diffraction where the number if grating elements defines the line-width: more grating lines narrower line-width.

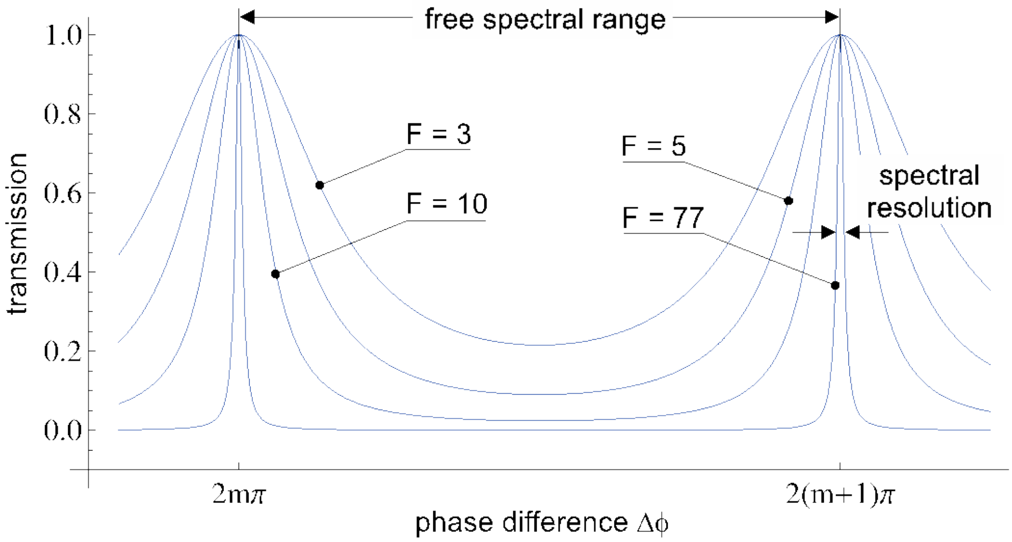

Figure 3: Normalized transmission of a Fabry-Pérot resonator for various values of the finesse F ≤ 77 as a function of the phase difference Δφ. A monochromatic light source is presumed.

Since all light rays slightly diverge from their source due to a limited spatial coherence, wavefronts will have a certain curvature and plane-mirror cavities always produce concentric interference rings rather than single on-axis spots. This rings are called "Haidinger" rings and are shown for illustration in the left of Fig. 4 for a source with pronounced spectral feature (deuterium lamp).

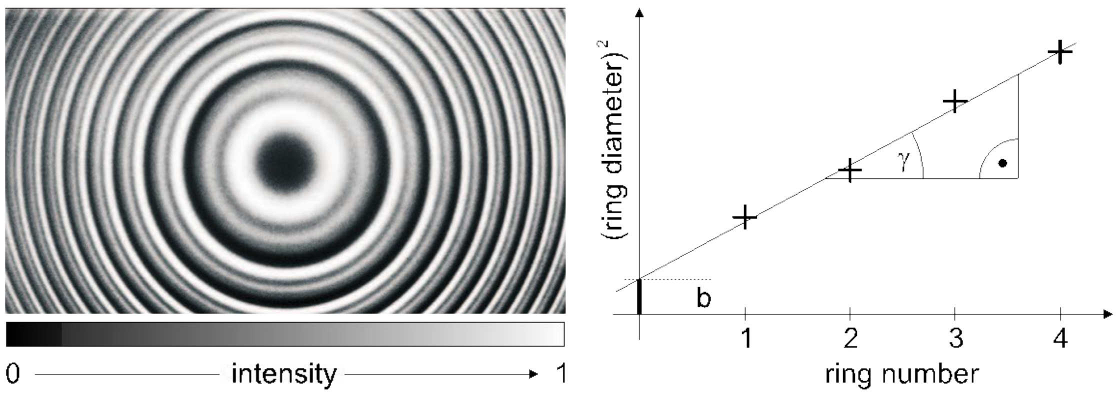

Figure 4: Concentric interference rings of a Fabry-Pérot cavity with plane mirrors for two closely spaced spectral lines (left) and the functional dependence of the squared diameters on the ring number (right). The left figure is taken from http://commons.wikimedia.org.

From their radial intensity distribution characteristic parameters of the setup may be obtained [2]. Following Fig. 2, the condition

$$2d\cos \alpha_p = m\lambda \qquad \text{with } p = 1,2,3,... \qquad (7)$$

leads to interference maxima for certain incidence angles α_p and integer numbers of m. Let the innermost ring be denoted by the index "0". We get from Equation (7) for the difference between the ring p and the center (0):

$$2d \cos \alpha_p - 2d \cos (\alpha_0) = (m - m_0)\, \lambda. \qquad (8)$$

In a real situation the parallel transmitted light is focused to the image plane by a lens with a focal length f. Thus, to get the diameter D_p of the p^{th} interference ring in the paraxial approximation on needs to calculate :

$$D_p = 2f\tan \alpha_p \approx 2f\alpha_p \qquad \text{for small angles } \alpha_p \rightarrow 0. \qquad (9)$$

Combining the Equations (8) and (9), we get a linear dependence of the squared diameter D_P^2 and the ring number p:

$$D^2 = 4f^2\, \lambda(p+\varepsilon)/d \qquad \text{with} \quad \varepsilon = \alpha_0^2 d/\lambda \qquad (10)$$

The quantity $0 \le \varepsilon < 1$ is called the "excess" and determines the axis intercept b on the right in Fig. 4. On the other hand, the slope tan γ of the function (10) yields the ratio λ/d.

3.3. The confocal resonator

The finesse as defined in (5) is valid for a *constant* cavity spacing d across its lateral dimensions. On-axis beams would thus receive the same optical path length as parallel rays that hit the system at different position (but the still at the same angle) when propagating through the resonator. In contrast, confocal arrangements as shown on the left of Fig. 5 are made of spherical mirrors and cause various path lengths depending on the distance of the ray from the optical axis. Such Fabry Perot systems can be much easily aligned. Beams which enter the confocal cavity with an off-axis parameter $\rho > 0$ (displacement from the axis for still parallel beams, see Fig. 5) would experience an optical path difference of $\rho^4/4r^3$, where r (do not mix up with reflection coefficient) is the radius of curvature of the mirrors (same radius of curvature for both mirrors)[1].

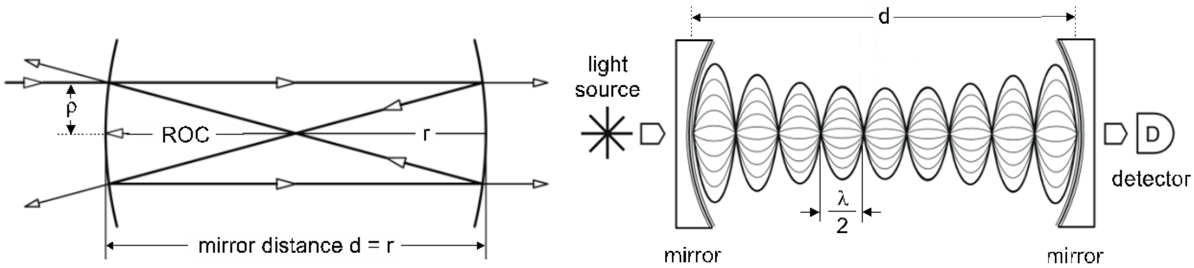

Figure 5: Left: Geometry of confocal cavities made of two identical spherical mirrors. The mirror distance d equals to the radius of curvature r. The parallel input beam is effectively transmitted without divergence. Figure is taken from [5]. Right: Amplitude of the electric field within the confocal resonator in the case of resonance. Figure is taken from Web sources.

In such a configuration, the finesse will be given by a more a general expression. One uses the total finesse F_t of confocal cavities and can write:

$$\frac{1}{F_t} = \sqrt{\left(\frac{1}{F_R}\right)^2 + \left(\frac{1}{F_q}\right)^2 + \left(\frac{1}{F_i}\right)^2} \quad \text{with} \quad F_q \to \infty \quad \text{and} \quad F_i = \frac{\lambda}{4}\frac{4r^3}{\varrho^4}, \tag{11}$$

where mirror irregularities (F_q) can often be neglected. The reflective term F_R limits the total finesse for perfectly aligned resonators and on-axis rays and the following relations holds: $F_t \leq F_R$. Figure 5 (right) gives a representation of the electric field amplitude within the resonator. It looks similar to a standing wave situation except the strong field enhancement due to multiple back and forth reflections.

Although the ray-optics approach is helpful in understanding the operating principle of Fabry-Perot cavities, it cannot provide any information about the spatial intensity distribution of the resonant modes. Those quantities can be determined by the use of wave-optics.

A particular beam can exist inside the cavity as a mode, if it is reproduced after a roundtrip. First we shall consider a Gaussian beam given by the complex electric field amplitude:

$$E(r,z) = E_0 \frac{W_0}{W(z)} \cdot \exp\left(-\frac{r^2}{W(z)^2}\right) \cdot \exp\left(-i\left[kz - \tan^{-1}\left(\frac{z}{z_0}\right) + \frac{k\rho^2}{2R(z)}\right]\right) \tag{12}$$

$W(z) = W_0\sqrt{1 + (z/z_0)^2}$, $\quad \rho^2 = x^2 + y^2$, $\quad k = 2\pi/\lambda$

The quantity z_0 is known as the Rayleigh range, at which distance the curvature of the field, R(z), acquires its maximum value. W_0 is the beam waist, the minimum value of the beam radius, W. The intensity at the transverse *x-y* plane and phase of the field are:

$I = I_0\left[\frac{W_0}{W(z)}\right]^2 \cdot exp\left[-2\left(\frac{\rho^2}{W^2(z)}\right)\right]$ and $\varphi(\rho,z) = kz - \zeta(z) + \frac{k\rho^2}{2R(z)}$, $\zeta(z) = \tan^{-1}\left(\frac{z}{z_0}\right)$, respectively.

In order for the field to retrace itself, the radius of curvature of the beam at the mirrors' position must be equal to the radii of curvature of the mirrors. The phase of a wave travelling on the optical axis ($\rho = 0$, the same conclusions can be drawn for every ρ, since the mirrors coincide with the wavefront) from mirror 1 at z_1 to mirror 2 at z_2 is:

$\varphi(0,z_2) - \varphi(0,z_1) = k(z_2 - z_1) - [\zeta(z_2) - \zeta(z_1)] = kd - \Delta\zeta$, $\quad \Delta\zeta = \zeta(z_2) - \zeta(z_1)$.

When the phase difference after a roundtrip is a multiple of 2π, the wave retraces itself and is a mode of the resonator. Enforcing this condition, we get:

$$\boldsymbol{2kd - 2\Delta\zeta = 2\pi q \Rightarrow \cdots \Rightarrow v_q = q\,\delta v + \frac{\Delta\zeta}{\pi}\,\delta v}. \tag{13}$$

Another family of beam profiles that can resonate inside the cavity are the Hermitian-Gaussian beams, since they have the same phase profile as a Gaussian beam, but different intensity profiles (Fig.6). The complex electric field amplitude of Hermite-Gaussian modes is given by:

$$E_{nm}(x,y,z) = E_0 \frac{W_0}{W(z)} \cdot H_l\left(\sqrt{2}\frac{x}{W(z)}\right) exp\left(-\frac{x^2}{W(z)^2}\right) \cdot H_m\left(\sqrt{2}\frac{y}{W(z)}\right) exp\left(-\frac{y^2}{W(z)^2}\right) \cdot$$

$$exp\left(-i\left[kz - (1+l+m)\cdot\zeta(z) + \frac{k\rho^2}{2R(z)}\right]\right),$$

Where H_k is a Hermitian polynomial of order k. The above formula makes it obvious that the intensity of the mode is the product of a Gaussian beam with a Hermitian polynomial, while the phase is the same as in the Gaussian case. Following the same procedure (and the same arguments) as before, the phase difference of the beam at $\rho = 0$ after a roundtrip is:

$$\boldsymbol{2kd - 2(l+m+1)\frac{\Delta\zeta}{\pi}\delta v = 2\pi q \Rightarrow \cdots \Rightarrow v_{l,m,q} = q\delta v + (l+m+1)\frac{\Delta\zeta}{\pi}\delta v} \qquad (14)$$

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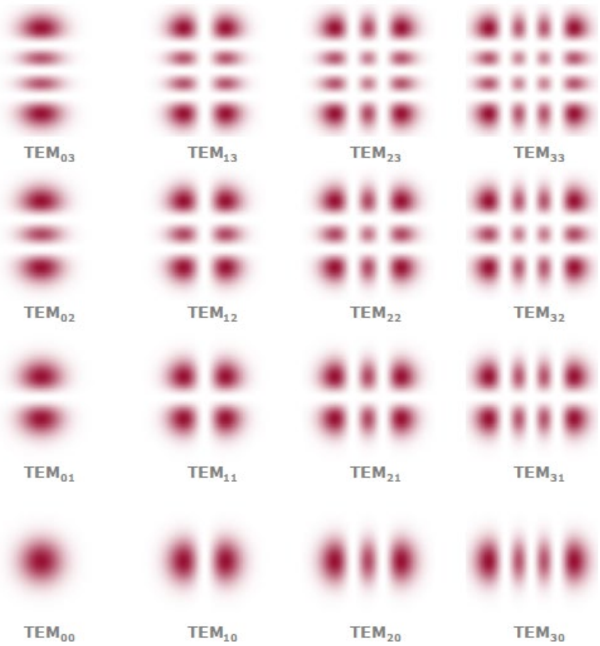

Figure 6: Intensity profile of the first 16 Hermite-Gaussian modes. Note that the $(0,0)$ mode collapses to the Gaussian mode described earlier. The first number gives the times the intensity drops to zero along the x-axis, while the second one gives the same quantity along the y-axis

According to Eq(14), we can differentiate between two types of modes:

1. **Longitudinal** or Axial modes, which have different q but the same (l,m) and thus identical intensity distribution
2. **Transverse** modes that have different (l,m)

Additionally, the same equation indicates the following properties:

- Longitudinal modes of a given transverse mode have δv free spectral range
- All transverse modes having the same sum of the indexes $(l+m)$ have the same resonant frequencies and are thus degenerated
- Two different transverse modes (l,m) & (l',m') that have the same q, have a spectral separation of:

$$v_{l,m,q} - v_{l',m',q} = [(l+m) - (l'+m')]\frac{\Delta\zeta}{\pi}\delta v$$

☖ Task 1: Answer the questions or be prepared to discuss the following subjects

Please, read up at home on the following keywords in common textbooks and / or the internet. You should be able to explain each of them with a few sentences and / or formulae:

1) Find an energy diagram with the different atomic levels for the He Ne laser! Can you explain the operation principle of a HeNe laser?
2) What are the modes in a resonator (cavity)?
3) What determines the number of modes in a resonator (cavity)?
4) How does a piezo actuator work?
5) What is the temporal coherence length and how is it measured?
6) Find a sensitivity curve of a silicon photo diode. Can you explain why there is a cut-off wavelength?
7) What is the design principle of a dichroic filter? Why one uses dichroic filters in multiple interference setups?

4. Experimental part

4.1. Setup and Equipment

We restrict the description to the most important parts of the kit which are essential for the experiments. Any additional information can be obtained from the original literature [3].

- With an output power of <5 mW, the **He-Ne laser** is classified as a class 3B product. The laser provides a temperature-dependent doublet emission line around 632.8 nm whose components are linearly polarized in perpendicular directions.
- Three pairs of **mirrors** are provided, two spherical sets with radii of curvature (ROC) of 75 mm and 100 mm, respectively; and one set of plane mirrors. The reflectivity of the mirrors is 96% for each set. (transmission T=4%)

Please, NEVER touch the mirror surfaces! The coatings are extremely damage- able and expensive. Id cleaning seems needed please ask the assistant.

In each mirror set, one of them (the smaller one) can be mounted on an axial **piezo translator**. The piezo translator contains a pre-stressed piezo-ceramic actuator. The piezo ceramic actuator is driven by a maximum voltage amplitude of 150 V and provides an open loop sensitivity of ≈ 0.05 nm for 5 mV noise and a maximum force generation of 5500 N. The following list overviews other technical features of the device:

Type	max. stroke	length L	capacitance	stiffness	resonance
HPSt 500/15 − 8/7	13/8 μm	26 mm	140 nF	550 N/μm	30 kHz

- The piezo actuator may be controlled with respect to amplitude, offset and frequency. It moves back and forth according to a periodical voltage profile applied. Figure 7 illustrates the front panel of the **control unit PTC 1000**. There are two voltage profiles: triangular and saw tooth. The high voltage connector (BNC) for the piezo is located on the back, as well as the piezo voltage monitor and the trigger signal output.
- The output signal of the silicon **photo diode** (Siemens BPX 61) is amplified using the same control unit PTC 1000. The photodetector has an active area of 2.65 × 2.65 mm^2 which is sufficient to collect even the unfocused laser light (FWHM □ 1 mm) without excessive losses. Its spectral range of sensitivity is given as 400 nm ≤ λ ≤ 1100 nm, with a maximum around 850 nm.

Although all measurements can be performed under daylight conditions, no direct sunlight or artificial room illumination should hit the photo diode in order to ensure an optimized contrast!

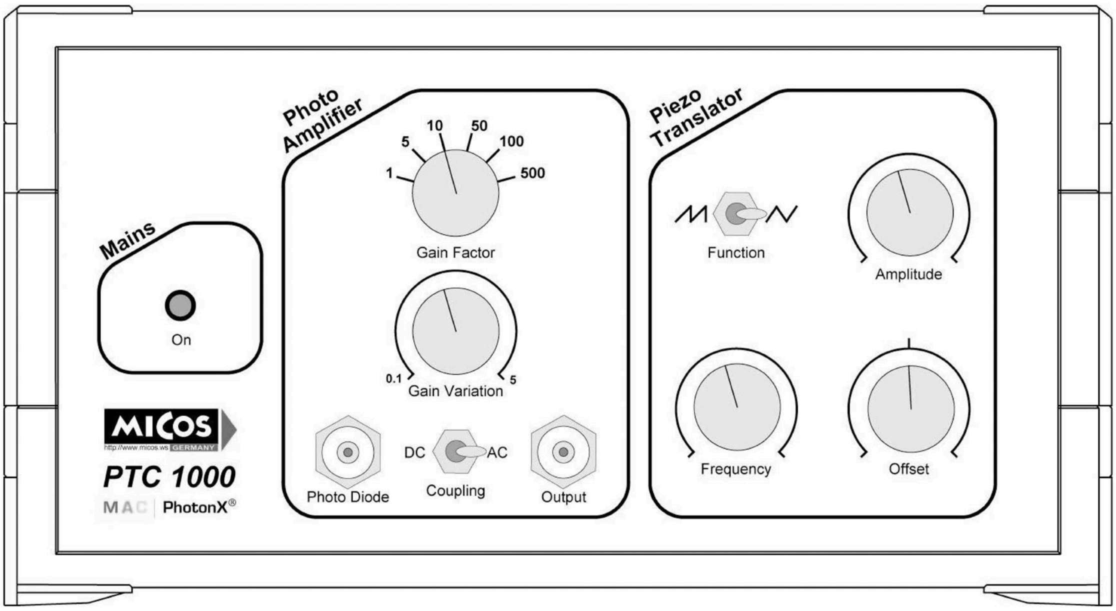

Figure 7: Front panel of the control unit PTC 1000. In the "photo amplifier" unit, the gain may be varied within 0.1 and 2500. The frequency of the piezo triangle (or saw tooth) voltage can be set between 50 Hz and 100 Hz. Figure is taken from [3].

- We use a Rohde Schwarz HMO2024 oscilloscope with a bandwidth of 200 MHz. The oscilloscope is triggered externally by the control unit PTC 1000 (back side connector). The amplified signal of the photo diode should be displayed on one channel, the second channel is reserved for the piezo triangle voltage. The front panel is shown in Fig. 8. An USB Flash drive with a capacity ≤ 2 GB may be used for quick storage of screen. The complete printed manual of the instrument is available in the lab room [4]. Please, don't remove it from the lab.
- For better sensitivity the light can be focussed on the detector. The output of the resonator is focused onto the photo diode using a **convex lens** with f = 60 mm.
- For the plane mirror arrangement, the laser beam should be expanded. One adjustable – **divergent lens** with f = −5 mm and two focusing ones with f = 20 mm (achromatic) and f = 150 mm are available.

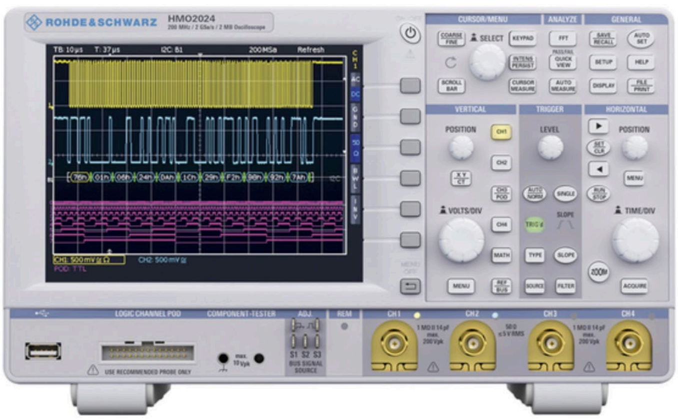

Figure 8: Front panel of the HMO2024. An additional USB port for connection with a PC (remote control) is located on the back of the instrument. Figure is taken from [4].

- A rotation stage equipped with a polarizer is available and can be used for more detailed measurements of the mode spectrum of the HeNe laser.

4.2. Confocal Arrangement for ROC = 75 mm

4.2.1. Adjustment

An appropriate adjustment of the resonator components along the optical axis is essential for successful measurements. In a first step, the confocal cavity with an ROC of 75 mm should be configured.

This standard setup can be adjusted quickly and is used for investigations of the mode spectrum of the laser and the determination of the piezo expansion. Figure 9 gives an overview of the configuration.

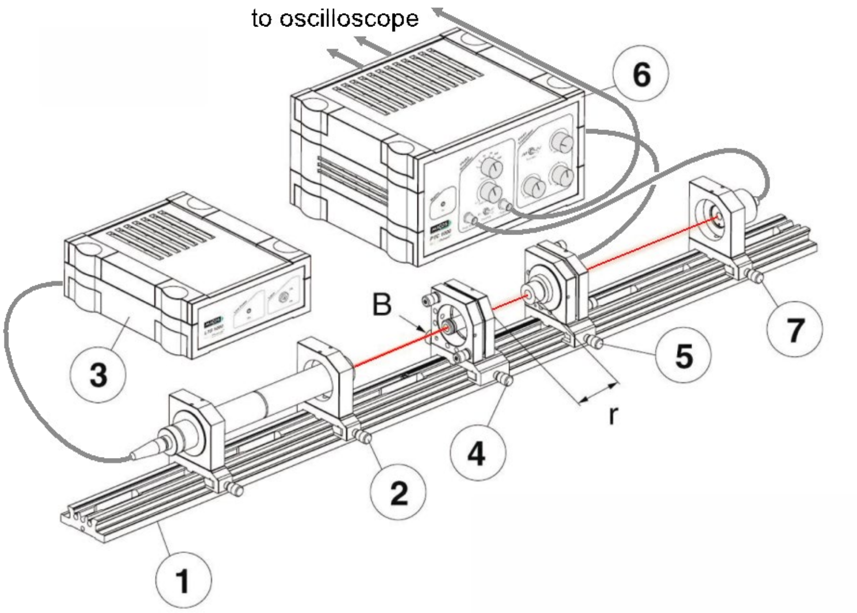

Figure 9: Setup of the Fabry-Pérot resonator with spherical mirrors in the confocal mode: (1) optical rail – (2) laser with mounting – (3) laser power supply – (4) fixed resonator mirror – (5) piezo-driven resonator mirror – (6) control unit – (7) photo diode. Figure is adopted from [3].

For first explorations, the photodiode amplifier should be set to DC coupling, for a gain of about 50 (“Gain Variation” → centre). It is reasonable to start with an ordinary triangle function for the piezo movement rather than the sawtooth profile. Choose an amplitude that is equivalent to a stroke near the maximum and medium values for frequency and offset. The recommended adjustment procedure is as follows (see Figure 10):

- Remove all tabs from the optical rail (1) except for the laser (2, 3).
- Mount the tab with the photo diode on the rail and put the amplified (6) signal of the photo diode on one channel of the oscilloscope.
- Insert the slider with the piezo stage with the mirror towards the laser (5 in Fig. 9). Adjust the mirror to back-reflect light towards the laser.
- Insert the slider with the second mirror (4 in Fig. 9). Adjust the mirror to back-reflect light towards the laser.
- Adjust again both mirrors (ROC = 75 mm) *separately* using back reflections and concentric interference rings.

- The mirror distance should be coarsely set to the confocal condition $d = r$ using the scale on the optical rail or a ruler
- Switch on the piezo translator, using initially a medium amplitude and frequency. The piezo voltage is put on the second channel of the oscilloscope.
- Change the distance between mirrors to adjust the system. It is important to find an efficient method for fine tuning of the mirror distance. Adjust until the interference contrast is maximized! For an optimized alignment, the finesse should approach the theoretical maximum.

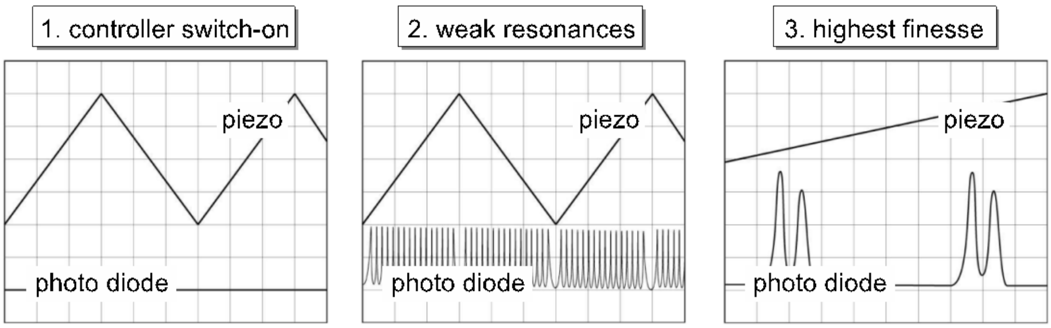

Figure 10: Adjustment of the (confocal) setup. The amplitude of the resonances should be scaled down by reducing the gain at the control unit and / or the "volts / div" knob of the oscilloscope. Figures are adopted from [3].

Using the CURSOR function of the oscilloscope, the distance between the resonance peaks and their width (FWHM) are measured now. Store appropriate screen shots on your memory stick for reference. Measure the mirror distance including its estimated error!

During the warm-up time (30 min- 1 h) of the laser tube, the mode spectrum of the HeNe emission around 632.8 nm varies. In particular, the relative intensities of the closely neighboured lines oscillate on time scales of several seconds. You should observe these variations until the equilibrium is reached. Now the *polarization* of the *final* peak intensities is measured for relevant angular positions $0^{\circ} \leq \varphi < 360^{\circ}$. Store the associated screen shots on your stick.

The piezo expansion rate is defined as the stroke in relation to the voltage difference and measured in units of [nm/V]. Check if it is independent of the piezo frequency and amplitude.

4.2.2. Measurements of confocal setup with ROC = 75 mm

Using the measured data for the peak width and their distance from each other, calculate the finesse as follows: Take an appropriate screen shot on the oscilloscope, similar to the right picture in Fig. 10. By means of the CURSOR function, measure the time distance $\delta\tau$ between two adjacent peaks of the same laser mode, corresponding to a phase shift $\Delta\varphi = 2\pi$. In the same way, determine the full width at half maximum (FHWM) $\Delta\tau$ for at least one of them or – better – take the mean FHWM of both peaks. The ratio $\delta\tau/\Delta\tau$ yields the dimensionless finesse F , though the latter one is defined by inverse units, i.e. frequencies.

Give results for the free spectral range and the spectral resolution of your setup based on the *measured* mirror distance *d* in units of [Hz] and estimate their errors: The FSR $\delta\nu$ is obtained from (6) and its follow-up comments. Roughly approximated, its experimental error originates from the uncertainty of *d* and should be given as $\delta\nu = \langle\delta\nu\rangle \pm \Delta(\delta\nu)$. The spectral resolution is calculated now by using equation (6) again, using the value for the finesse F from above. Its "± error" should be calculated in a similar way as for $\delta\nu$.

Compare your results with theoretical predictions from the mirror's reflectivity and their ROC: Use (5) and (6) once more, now for the nominal data provided by the manufacturer.

Determine the wavelength separation of the two adjacent laser emission lines and discuss their polarization properties: Note that you need to convert the directly measurable time difference from the oscilloscope into wavelengths. There is no linear relation as in case of the finesse! The polarization features should be documented by a couple of printed screen shots for various angular positions of the polarizer.

Calculate the piezo expansion rate in units of [nm/V] from the oscilloscope screen shot: Consider what happens during piezo expansion: How are the observed resonance peaks of the Fabry-Pérot related to certain mechanical positions of the piezo? Use an oscilloscope display similar to the right in Fig. 10.

4.3. Stable Configurations for ROC = 100 mm

According to Section 3, the free spectral range (FSR) and the spectral resolution depend on the mirror distance d. Replace the spherical mirrors with r = 75 mm by those with an ROC of 100 mm and adjust the system again as described above. The distance between the resonance peaks and their width (FWHM) are measured again using the CURSOR. Store appropriate screen shots on your memory stick and estimate the real mirror distance using the scale on the optical bench.

The *stability* of an optical resonator depends on the geometry of the system. Aside from confocal arrangements, several other configurations allow stable operation. The general condition for a stable cavity is given as

$$0 \le (1 - d/r_1) \cdot (1 - d/r_2) \le 1 \tag{15}$$

for two mirrors with radii of curvature r_1 and r_2. This equation is illustrated in Fig. 11. For example, the concentric cavity is defined by $d = 2r$.

Expand the mirror distance according to this condition and optimize the interference contrast. It might be helpful to use an additional focusing lens (e.g. f = 60 mm) in front of the photo diode. Record all required data and screen shots for the determination of the finesse as before.

Determine the finesse using the oscilloscope data. Calculate the free spectral range and the spectral resolution of this configuration as before and discuss your results: Obviously, the procedure is the same as for the "75 mm" case. Check if your data yield reasonable results.

There are several configurations as explained in literature [2] and the one with a mirror distance equal to the double of the mirrors radios of curvature is called concentric. Usually, the concentric resonator performs worse than the confocal one. Be careful to get a regular resonance pattern as shown on the right in Fig. 10.

Copy the stability plot (Fig. 11) into your lab report and mark the corresponding points for the confocal and the concentric case. Calculate the stability criterion for all other combinations of mirrors used in this lab, i.e. the spherical ones with an ROC of 75 mm and 100 mm, respectively, and the plane mirrors ($r \rightarrow \infty$). In this way, you will obtain 6 possible resonator designs. Identify and discuss their stability in your plot.

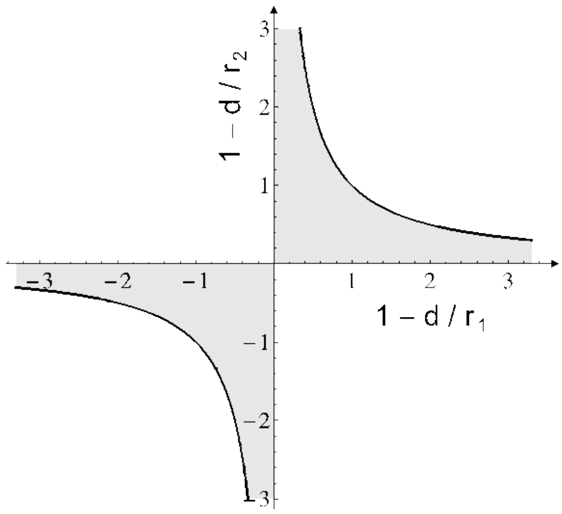

Figure 11: Stability of optical resonators made of two mirrors with radii of curvature r1 and r2, separated by a distance d. The stable regions are gray shaded areas and include the central point.

4.4. Plane-Mirror Cavity

4.4.1. Adjustments and setup

This experiment uses two plane resonator mirrors instead of spherical ones. It is the most challenging arrangement. An optimized detection of the interference rings requires a new experimental approach. Remove all optical mounts except for the laser.

- In a first step, the laser beam should be expanded with a two-lens telescope: It is recommended to use the f = 150 mm lens rather than the device with f = 20 mm in conjunction with the divergent lens (f = −5 mm). The output beam must have a constant diameter, which is independent of the distance from the beam expander.
- Place the piezo-mounted mirror on the optical bench and adjust the back-reflected beam as accurate as possible. Switch off the control unit PTC 1000 during the adjustment.
- Think about a reasonable mirror distance d. On the one hand, the sensitivity of the setup on misaligned mirror surfaces increases with the distance d. Thus, the adjustment is less challenging for a short mirror distance. On the other hand, the spectral resolution becomes better for large values of d.
- Place the second mirror on the bench and adjust it coarsely using back-reflections.
- Install the CMOS camera at the end of the optical bench and the convex lens (f = 60 mm) in front of it – somewhat different like in Fig. 12 but now component 8 is the camera. Display the transmitted spot on the PC screen and centre the CMOS camera to the optical axis.
- Think about an efficient “algorithm” for fine tuning of the cavity! Exploit the concentric fringe pattern which is shown on the PC screen (as on the left of Fig. 4).
- Set the polarizer into the laser beam and try to optimize the symmetry, sharpness and interference contrast of these so-called Haidinger rings. Exploit the polarizer for investigations of the mode spectrum, if applicable.

- Store at least one adequate image in the .bmp format on your memory stick – you may also record a cross section of the radial intensity distribution through the centre of the ring pattern: The camera software offers an option for visualizing such one- dimensional cross sections in horizontal and / or vertical direction. You should get a screenshot by the combination□*ALT GR*□ □*PRINT*□and copy it from the clipboard into an empty work sheet of an appropriate software and store the file on your stick.
- Note your individual mirror separation d and estimate its uncertainty, i.e. $d \pm \Delta d$.
  Download the CMOS camera manual [6] from the lab PC to your memory stick. You will need this reference for information on the sensor and pixel size of the camera. The model in use is called "DCC1545M".
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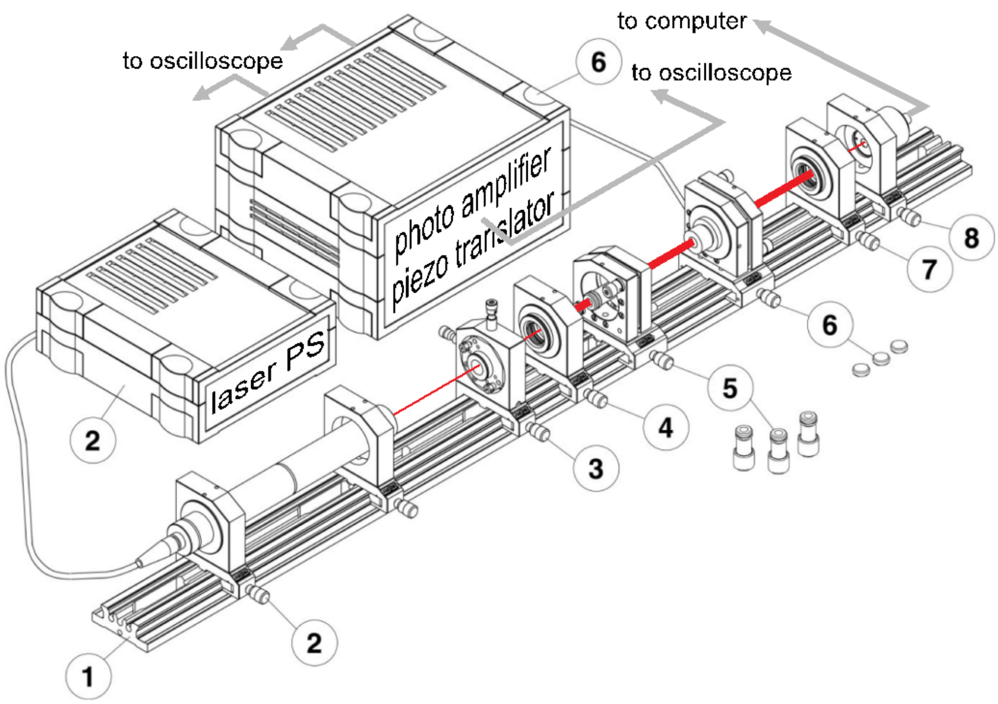

Figure 12: Setup of the Fabry-Pérot resonator with plane mirrors: (1) optical rail – (2) laser with mounting and power supply – (3)+(4) beam expander – (5) fixed mirror – (6) piezo-driven mirror and control unit – (7) focusing lens – (8) CMOS camera. Figure is adopted from [3].

4.4.2. Characterization of concentric setup

Calculate the mirror distance and its standard deviation using the concentric ring system and compare your result with the directly measured value. Give the value of the excess ε: The procedure is described in Sect. 3, in particular by Fig. 4 and Eq. (10). At first, you should determine the diameters of at least three rings – don't confuse about the two laser modes – using a print-out of the CMOS image or by image-processing of its .bmp version.

For this purpose, the three-dimensional RGB data of the .bmp file must be first converted to one-dimensional gray-scale values. Depending on your software, this is done by a single command as in Mathematica for instance or alternatively in Photoshop or other software tools. Afterwards, the central row or column through the fringe pattern has to be extracted.

Plot your values for D_p^2 versa the ring number p and add a linear least-squares fit to the data. If possible, take an appropriate software (Matlab, Mathematica, Maple,...) or an advanced scientific pocket calculator for this task for the sake of accuracy. Note that an excess $\varepsilon < 0$ indicates an error in your analysis! So you should consider carefully the nature of the central intensity distribution, i.e. if it is indeed a single spot or a local minimum.

If you have recorded fringe patterns for both polarizations, you might even perform the procedure for those two adjacent modes and try to find their wavelength separation. Since the difference in the slope is very small, this operation works probably only for a sufficiently large number of detected rings and thus a small statistical uncertainty in the linear fit.

Calculate the interference order for this setup and deduce the FSR in units of [Hz]: As described by Eq. (4) and Fig. 5, the interference order is related to the wavelength λ and the mirror distance *d*. A modified version of this relation is valid for diffraction gratings, too. Why? If you want to measure the *absolute* wavelength of an unknown light source, would you either use a Fabry-Pérot resonator or that grating for spectroscopy?

Plot the radial cross section of the intensity distribution $I(r)$ and estimate the visibility as a function of the radius: There are two options for drawing $I(r)$. Either you may use the built-in function from the CMOS camera software and take a screen shot as described in Sect. 4.1.3. Alternatively, mathematical software tools extract the corresponding data row in the gray-level encoded matrix of the .bmp file. Once plotted, the peaks and the minima in between should be marked and measured both in radius and intensity. An example is illustrated in Fig. 13. The visibility is defined as $V(r) \equiv (I_{\max}(r) - I_{\min}(r))/(I_{\max}(r) + I_{\min}(r))$.

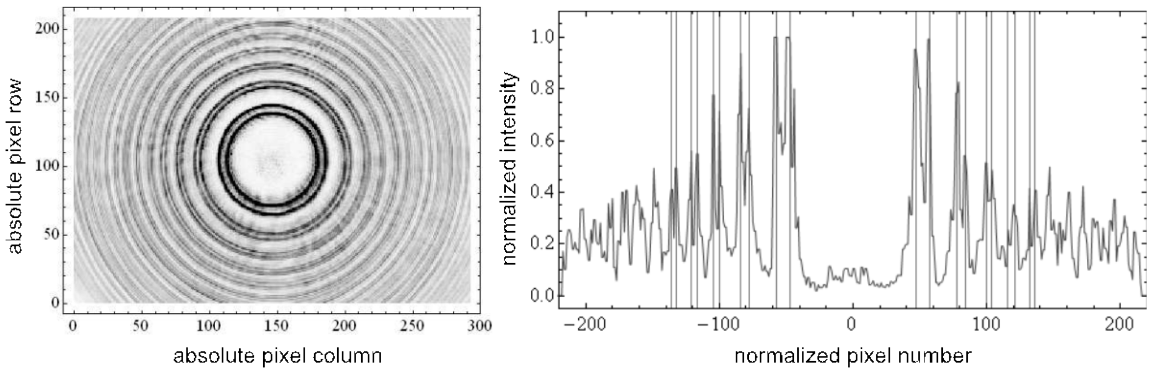

Figure 13: Analysis of the plane-mirror fringe pattern. On the left, the .bmp pixel image is shown. On the right, a cross section through the center of the ring system reveals the peaks and minima.

5. Final questions

In addition, you should complete your lab report by solving the following problems. Their intention is to get an overview on the subject and a deeper understanding.

- In Sect. 3, the high spectral resolution of a Fabry-Pérot device was explained by multiple beam interference. Estimate the number of rays for an interferometer with a finesse F . Compare your result with the number N of lines for a typical grating, whose resolving power in the m[th] order is given as $\lambda/\Delta\lambda = mN$. How do Fabry-Pérot resonators achieve their spectral resolution?

- Give a reason why the free spectral range of a plane mirror cavity is always $c/(2d)$ whereas the FSR for confocal geometries may be just one half, i.e. $c/(4d)$ for pronounced off-axis beams. What is the FSR for concentric arrangements (d = 2r)?
- Calculate and plot the function $F_t(\rho)$ from Equation (11) for a mirror reflectivity of 96%. Why we don't use the beam expander for the confocal geometry?
- Lasers often use spherical mirrors in the confocal distance (d = r). Why is this arrangement so important?
- An important feature of optical resonators is their "stability". Scan the literature and / or the web in order to find out how the criterion from Section 4.3 is derived – at least in principle. Is it a "yes/no" criterion or may the stability be quantified?
- The plane mirror mode gives reason for simple investigations of the mirror surface quality. Unlike spherical ones, imperfect plane mirrors cause serious wavefront errors after multiple back and forth reflections, since even tiny phase shifts induced by small deviations from the ideal plane surface add up and smear out the interference contrast. The surface roughness is often measured in fractions of the wavelength, $\Delta h = \lambda/n$. Values $\Delta h < \lambda/100$ cannot be realized without an extraordinary technical effort. As a consequence, the finesse is usually limited to F = 50, even for high reflecting mirrors. Fig. 14 compares both limits. Give an explanation for the functional dependence on the right of Fig. 14. Following this elementary theory, you can estimate the required surface quality of the mirrors used in our lab for an optimized finesse near the theoretical limit.
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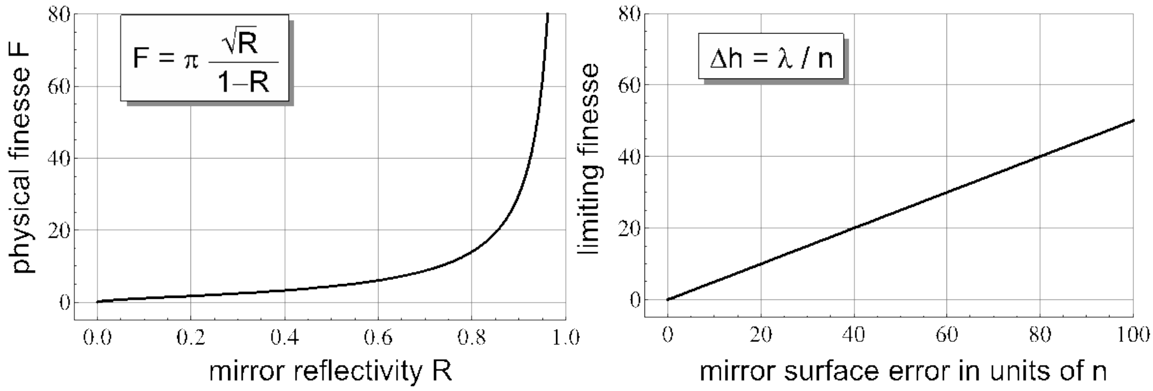

Figure 14: The physical and technical finesse for plane mirrors. On the left, the reflection-based finesse F is shown. The technical limit is given in units of the wavelength fraction n (right).

- Reconsider the concentric ring pattern in the plane mirror arrangement. Should the mirror distance d be enlarged or diminished in order to contract the ring diameters?
- Following its definition in Sect. 4.4.2, the visibility V describes the interference contrast. Find the functional dependence V(F) on the finesse.
- Why did we use the polarizer and a CMOS camera for the plane-mirror experiment? Based on the measured wavelength distance for the two-mode spectrum, estimate if they could have been resolved by *your* plane-mirror setup. Reason your statement!

6. Preparation of the report

- Please keep the theory part as short as possible, about 1 − 2 pages with the most important concepts and equations. You should not just repeat the manual!
- Work through Sect. ***Experiments*** step by step and try to answer all questions. Please include your answers to the preliminary (A) and final (B) questions, too.
- Describe in detail what you are measuring; i.e. all *original* plots and data must be provided. When you calculate your results, describe the way you found them (derivation of formulae).
- Make sure that all results are given with correct units. That is for instance to use [Hz] or s^{-1} for frequencies, not [s] or something else.
- Discuss your results, in particular in case of mismatching or contradictory results. If you were not able to do certain parts of the work program, please explain!

7. References

[1] http://www.thorlabs.com,"ScanningFabry-PérotInterferometers", Website.

[2] W. Demtröder, "Laser Spectroscopy Vol. 1: – Basic Principles", Springer-Verlag, Berlin/Heidelberg, 5th edition, 151 ff (2014). W. Lauterborn, T Kurz - Coherent Optics – Springer Berlin 1993 Chapter 5 Multiple beam interference.

[3] eLas GmbH, "Laser Education Kit – CA-1140 Fabry Perot Resonator", user manual, Eschbach (2009).

[4] Rohde Schwarz, Digital Storage Oscilloscopes HMO 2024: User Manual"

[5] MEOS Dr. W. Luhs MEOS GmbH, "Fabry Perot Resonator", Eschbach (2003).

[6] ThorlabsGmbH, OperationManual"High-ResolutionUSB2.0CMOSandCCDCam- eras", 85221 Dachau, Germany (2009).